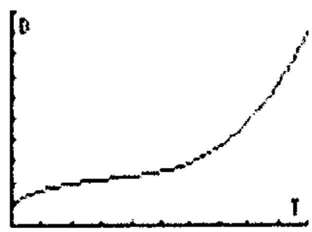
The Fundamental Theorem of Calculus

The objective of this lesson, is to discover how to evaluate definite integrals analytically, without our calculator. This result is called the **Fundamental Theorem of Calculus** (FTC).

Let's develop the FTC through an "application".

Honors Physics student Kal Kulis is performing an experiment. He has set a motion detector on a table and it is recording his distance in feet from the motion detector as he is walking away. He collects this data over a 10 second period, enters it into his calculator, and graphs it. The result is shown below.



The graph shows a Distance vs. Time graph. Kal, being also an excellent mathematics student, wants to find a function that models the graph. He decides that the data might be modeled by a cubic function, so he performs a cubic regression on his calculator, and gets a perfect fit!

His function, which represents his distance s(t) from the motion detector at any time t is:

$$s(t) = \frac{1}{10}t^3 - t^2 + 4t + 5$$

Questions:

1. How far did Kal walk over the 10 second time interval? ______________________________

2. Recall that a velocity function is the derivative of a distance function. Sketch a graph of the velocity function below.

3. What is the velocity function for Kal's distance function $s(t) = \frac{1}{10}t^3 - t^2 + 4t + 5$?

$v(t) =$ ______________________________

4. Since the velocity function is a "rate of change" function, how could we use it to determine how far Kal walked over the 10 second time interval?

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Let's calculate it and see what we get! ______________________________

So, here's what we just discovered.

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And, since the velocity v(t) is the derivative of the distance s(t), then the distance s(t) is the ______________ of the velocity v(t).

Let's generalize this result.

If F(x) is the antiderivative of f(x), and we are asked to evaluate $\int_a^b f(x)dx$, we can do this by writing

$$\int_a^b f(x)dx = \underline{\hspace{10cm}}$$

This is called the **Fundamental Theorem of Calculus**, and we can use it to evaluate definite integrals, if we know how to find the antiderivative of the integrand.

Examples and Problems

1. Earlier we used the definition of the definite integral, and our calculator, to show that:

   a. $\int_1^3 (16-x^2)dx = 23.333...$ Verify this.

   b. $\int_0^\pi \sin x dx = 2$ Verify this.

2. Use the Fundamental Theorem of Calculus to evaluate the following definite integrals. Check your answers with your calculator.

   a. $\int_1^9 \sqrt{x}\, dx$

   b. $\int_0^1 (5e^x - 6x)\, dx$